PASCAL'S TRIANGLE

100% of the numbers are even!

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Pascal's triangle is an arrangement of the binomial coefficients in a triangle. Each number inside Pascal's triangle is calculated by adding the two numbers above it. Figure 1 shows the first five rows of an infinite number of rows.

When all the odd integers in Pascal's triangle are highlighted (black) and the remaining evens are left blank (white), one of many patterns in Pascal's triangle is displayed (Figure 2).

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Figure 1

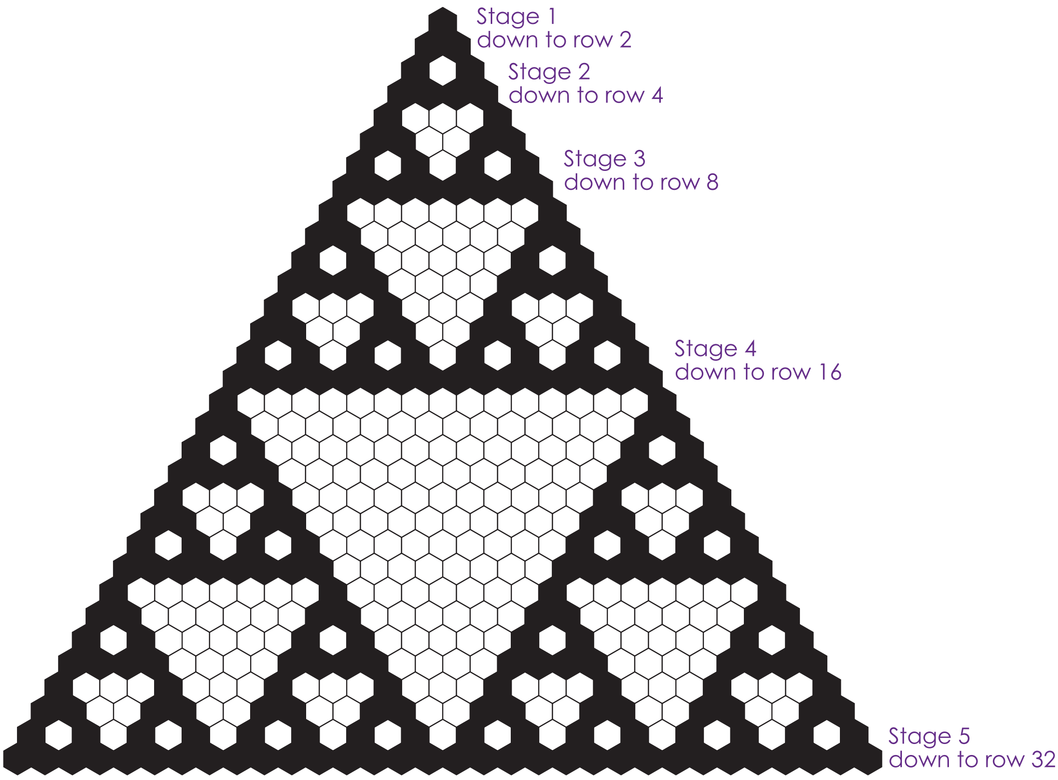

Figure 2

By comparing the pattern of black cells (odd integers) to the shaded parts of the fractal called the Sierpinski triangle, we were guided to the conjecture that as the number of rows of Pascal's triangle increases, so too does the percentage of even numbers, i.e., the further down you look, the whiter the pattern seems to get.

We decided to consider the triangle as a number of stages, where each stage finishes at a row of odd numbers as shown in the diagram above.

It is clear that each stage is built up from previous stages. For example,

- Stage 4 is made up of Stage 3 repeated 3 times with the hole in the middle white;
- Stage 5 is made up of Stage 4 repeated 3 times with the hole in the middle white;
- Stage 6 is made up of Stage 5 repeated 3 times with the hole in the middle white; and so on.

From this, we calculated the percentage of even numbers in each stage up to the third.

Stage 1

1
1 1

3 odd numbers, 0 even numbers

Percentage evens to odds $= \frac{0}{3} = 0\%$

Stage 2

1
1 1
1 2 1
1 3 3 1

9 odd numbers, 1 even number

Percentage evens to odds $= \frac{1}{10} = 10\%$

Stage 3

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

27 odd numbers, 9 even numbers

Percentage evens to odds $= \frac{9}{36} = 25\%$

The following (Table 1) shows our results in tabulated form.

Table 1

Stage	No. even	No. odd	Total	% even	% odd
1	0	3	3	0	100
2	1	9	10	10	90
3	9	27	36	25	75
n	?	?	?	?	?

Following this, we set out to find what percentage of integers would be even in the nth Stage.

Each stage ends when all the numbers on the bottom row are odd. This plays a big part in the formulas for working out the number of odds and the total number of numbers in Pascal's triangle.

We found that if you raise 3 to the power of the stage number, it would give you the number of odds down to that stage:

e.g., In the first stage the number of odds is $3^1 = 3$

In the second stage the number of odds is $3^2 = 9$

In the third stage the number of odds is $3^3 = 27$

So the number of odd numbers in the nth stage will be 3^n.

We then saw that the total number of numbers in different stages could be figured out using an arithmetic series:

$$\text{Sum} = \frac{(a+l)n}{2}$$

where a = the first number

l = the last number

n = the number of numbers

For example, the number of numbers in

Stage 2 is 1 + 2 + 3 + 4 $= \frac{(1+4)4}{2} = \frac{(1+2^2)2^2}{2} = 10$

Stage 3 total $= \frac{(1+8)8}{2} = \frac{(1+2^3)2^3}{2} = 36$

Stage 4 total $= \frac{(1+16)16}{2} = \frac{(1+2^4)2^4}{2} = 136$

Since this formula works for every stage, the number of numbers in the nth stage is:

$$\frac{(1+2^n)2^n}{2}$$

So the fraction of odd numbers in Pascal's triangle to the nth stage

$$\frac{3^n}{\frac{(1+2^n)2^n}{2}}$$

$$= \frac{2\times 3^n}{(1+2^n)2^n}$$

$$= \frac{2\times 3^n}{2^n + 4^n}$$

$$= \frac{200}{\left(\frac{4}{3}\right)^n + \left(\frac{2}{3}\right)^n}$$

Table 2 shows the values of this function for n values up to 20.

Table 2

n	% odd	% even	n	% odd	% even
1	100	0	11	8.4	91.6
2	90	10	12	6.3	93.7
3	75	25	13	4.8	95.2
4	59.6	40.4	14	3.6	96.4
5	46	54	15	2.7	97.3
6	35	65	16	2	98
7	26.5	73.5	17	1.5	98.5
8	19.9	80.1	18	1.1	98.9
9	15	85	19	0.8	99.2
10	11.3	88.7	20	0.6	99.4

The percentage of odd numbers clearly becomes very low and the percentage of even numbers seems to approach 100.

If we look at the formula and let $n \to \infty$,

$$\begin{aligned}\text{Percentage odd} &= \frac{200}{\left(\frac{4}{3}\right)^{\infty} + \left(\frac{2}{3}\right)^{\infty}}\\ &= \frac{200}{\infty + 0}\\ &= 0\end{aligned}$$

So 100% of the numbers are even, (although they are not)!

From Helen Prochazka's

Scrapbook

Facebook and the White House

In June 2009, Vivek Kundra, the US Chief Information Officer, posted the following on Facebook to publicise a project that will make stores of government data freely available.

The White House's Notes

Data Transparency via Data.gov

Government data permeates our lives. The atomic clock at the National Institute of Standards and Technology (NIST) standardizes our time, dictating when we arrive at meetings and take our children to soccer practice. The Centers for Disease Control and Prevention (CDC) provides our doctors and media outlets with information about how to keep our families healthy when there is a new public health concern, such as the H1N1 (swine flu) virus.

Data is powerful. It informs and it creates opportunities. It promotes transparency and it helps to ensure accountability. Yet, it is a challenge to collect, organize, and communicate the vast stores of data maintained across the government.

The Administration is committed to moving past these barriers in providing the American public with unprecedented access to useful, unfiltered government data. An important part of that effort is Data.gov, a platform for free access to data generated across all federal agencies. Through Data.gov, we aim to provide an open architecture and to make data available in multiple formats. The goal of Data.gov is to enable better decision-making, drive transparency, and help to power innovation for a stronger America. If you haven't yet checked it out, I encourage you to do so. Whether for a school research project, developing a new application, or evaluating a business opportunity, you might just be surprised by what you find.